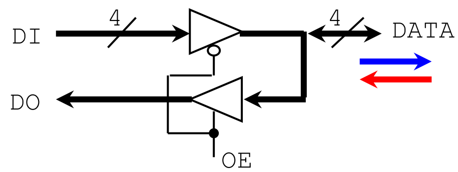
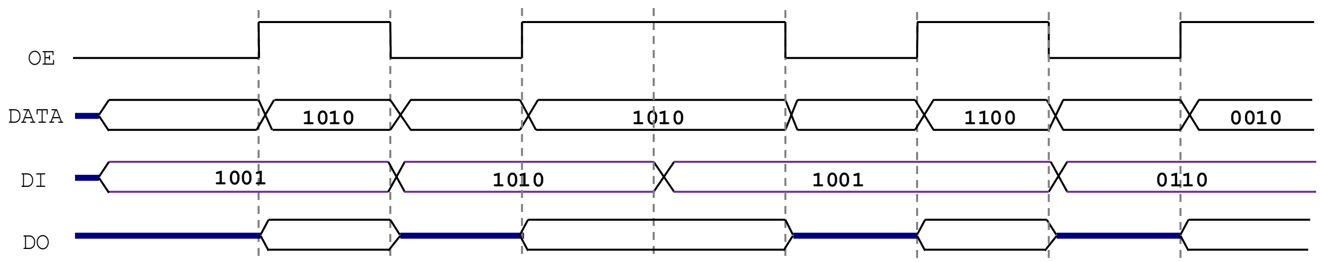
Homework 2

(Due date: February 4th @ 5:30 pm)
Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (5 PTS)

- For the following 4-bit bidirectional port, complete the timing diagram (signals *DO* and *DATA*):

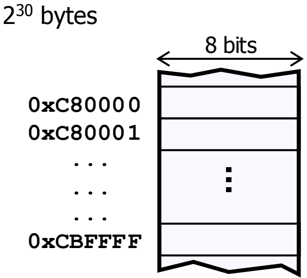




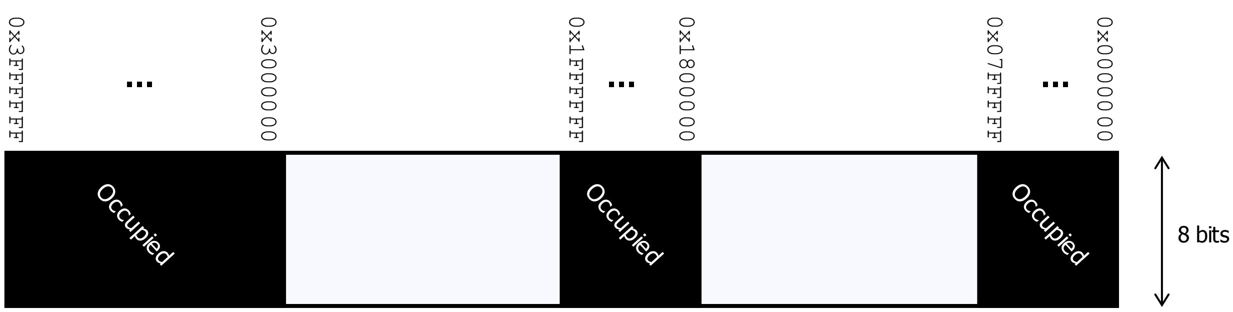
PROBLEM 2 (15 PTS)

a) What is the minimum number of bits required to represent: (2 pts)
  ✓ 220,000 colors? ✓ Numbers between 65,000 and 69,096?

b) A microprocessor has a 24-bit address line. The size of the memory contents of each address is 8 bits. The memory space is defined as the collection of memory positions the processor can address. (5 pts)
- What is the address range (lowest to highest, in hexadecimal) of the memory space for this microprocessor? What is the size (in bytes, KB, or MB) of the memory space? 1KB = 2^{10} bytes, 1MB = 2^{20} bytes, 1GB = 2^{30} bytes
- A memory device is connected to the microprocessor. Based on the size of the memory, the microprocessor has assigned the addresses `0xC80000` to `0xCBFFFF` to this memory device.
  - What is the size (in bytes, KB, or MB) of this memory device?
  - What is the minimum number of bits required to represent the addresses only for this memory device?



c) The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. (8 pts)
- What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor?
- If we have a memory chip of 8MB, how many bits do we require to address 8MB of memory?
- We want to connect the 8MB memory chip to the microprocessor. Provide an address range so that 8MB of memory is properly addressed. You can only use any of the non-occupied portions of the memory space as shown below.



PROBLEM 3 (20 PTS)

- In these problems, you MUST show your conversion procedure. **No procedure ≡ zero points**.
  a) Convert the following decimal numbers to their 2's complement representations: binary and hexadecimal. (6 pts)
  ✓ `-93.3125, 172.65625, -64.5078125, -71.25.`

  b) Complete the following table. The decimal numbers are unsigned: (8 pts.)

Decimal	BCD	Binary	Reflective Gray Code
299			
		1001001011	
			10100101010
128			
			11110101
		1110010	
			101001000
	100000011001		

  c) Complete the following table. Use the fewest number of bits in each case: (6 pts.)

	REPRESENTATION		
Decimal	**Sign-and-magnitude**	**1's complement**	**2's complement**
-126			
			10011001
		10111001	
			1000000000
	1100111		
		011010011	

PROBLEM 4 (30 PTS)

a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits n to represent both operators. Indicate every carry (or borrow) from c_0 to c_n (or b_0 to b_n). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher bit. (8 pts)

Example (n=8):

✓ `54 + 210`

$c_8=1$, $c_7=1$, $c_6=1$, $c_5=1$, $c_4=0$, $c_3=1$, $c_2=1$, $c_1=0$, $c_0=0$

```
 54 = 0x36 =  0 0 1 1 0 1 1 0 +
210 = 0xD2 =  1 1 0 1 0 0 1 0
---------------------------------
Overflow! → 1 0 0 0 0 1 0 0 0
```

✓ `77 - 194`

Borrow out! → $b_8=1$, $b_7=0$, $b_6=0$, $b_5=0$, $b_4=0$, $b_3=0$, $b_2=1$, $b_1=0$, $b_0=0$

```
 77 = 0x4D =  0 1 0 0 1 1 0 1 -
194 = 0xC2 =  1 1 0 0 0 0 1 0
---------------------------------
              0 0 0 0 1 0 1 1
```

✓ `221 + 117`
✓ `76 + 175`
✓ `93 - 128`
✓ `130 - 43`

b) We need to perform the following operations, where numbers are represented in 2's complement: (16 pts)
✓ `-93 + 128`
✓ `312 + 718`
✓ `87 - 62`
✓ `-255 - 69`

- For each case:
  ✓ Determine the minimum number of bits required to represent both summands. You might need to sign-extend one of the summands, since for proper summation, both summands must have the same number of bits.
  ✓ Perform the binary addition in 2's complement arithmetic. The result must have the same number of bits as the summands.
  ✓ Determine whether there is overflow by:
    i. Using c_n, c_{n-1} (carries).
    ii. Performing the operation in the decimal system and checking whether the result is within the allowed range for n bits, where n is the minimum number of bits for the summands.
  ✓ If we want to avoid overflow, what is the minimum number of bits required to represent both the summands and the result?

c) Perform the multiplication of the following numbers that are represented in 2's complement arithmetic with 4 bits. (6 pts)
✓ `0101×0101, 1000×0111, 0111×1001.`

PROBLEM 5 (10 PTS)

- Given two 4-bit signed (2's complement) numbers A, B, sketch the circuit that computes $|A - 2 \times B|$. You can only use full adders and logic gates. Make sure your circuit works for all cases. If there is overflow, design your circuit so that the final answer is always the correct one with the correct number of bits (i.e., overflow must be avoided).

PROBLEM 6 (20 PTS)

a) Implement the following functions using i) decoders (and OR gates) and ii) multiplexers: (5 pts)
   ✓ $F_a = (\overline{Y + Z}) \oplus (XY)$ ✓ $F_b = (\overline{\bar{X} \oplus Y}) \oplus \bar{Z}$

b) Using only a 4-to-1 MUX (do not use NOT gates), implement the following functions. (5 pts)
   ✓ $F_a(X, Y, Z) = \sum(m_0, m_2, m_4, m_6, m_7)$ ✓ $F_b(X, Y, Z) = \prod(M_2, M_4, M_6)$

c) Complete the timing diagram of the circuit shown below. Note that $x = x_1 x_0, y = y_3 y_2 y_1 y_0$ (10 pts)